On the Payoff Mechanism in Peer-Assisted Services with Multiple Content Providers

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Abstract

This paper studies an incentive structure for cooperation and its stability in peer-assisted services when there exists multiple content providers, using a coalition game theoretic approach. We first consider a generalized coalition structure consisting of multiple providers with many assisting peers, where peers assist providers to reduce the operational cost in content distribution. To distribute the profit from cost reduction to players (i.e., providers and peers), we then establish a generalized formula for individual payoffs when a “Shapley-like” payoff mechanism is adopted. We show that the grand coalition is unstable, even when the operational cost functions are concave, which is in sharp contrast to the recently studied case of a single provider where the grand coalition is stable. We also show that irrespective of stability of the grand coalition, there always exist coalition structures which are not convergent to the grand coalition. Our results give us an important insight that a provider does not tend to cooperate with other providers in peer-assisted services, and be separated from them. To further study the case of the separated providers, three examples are presented: (i) Each peer is underpaid than his due payoff, (ii) a service monopoly is possible, and (iii) the peer payoffs based on the Shapley-like mechanism exhibit even oscillatory behaviors. Analytical studies and examples in this paper open many new questions such as realistic and efficient incentive structures and the tradeoffs between fairness and individual providers’ competition in peer-assisted services.

I. INTRODUCTION

The Internet is becoming more content-oriented, and cost-effective and scalable distribution of contents has been the central role of the Internet. Uncoordinated peer-to-peer (P2P) systems, e.g., BitTorrent, has been successful in distributing contents, but the rights of the content owners are not protected well, and most of the P2P contents are in fact illegal. In its response, a new type of service, called peer-assisted services, has received significant attentions these days. In peer-assisted services, users commit a part of their resources to assist content providers in content distribution with objective of enjoying both scalability/efficiency in P2P systems and controllability in client-server systems. Examples of application of peer-assisted services include nano data center [1] and IPTV [2], where high potential of operational cost reduction was observed. However, it is clear that most users will not just “donate” their resources to content providers. Thus, the key factor to the success of peer-assisted services is how to (economically) incentivize users to commit their valuable resources and participate in the service.

One of nice mathematical tools to study incentive-compatibility of peer-assisted services is the coalition game theory which covers how payoffs should be distributed and whether such a payoff scheme can be executed by rational individuals or not. In peer-assisted services, the “symbiosis” between providers and peers are sustained when (i) the offered payoff scheme guarantees fair assessment of players’ contribution under a provider-peer coalition and (ii) each individual has no incentive to exit from the coalition. In the coalition game theory, the notions of Shapley value and the core have been popularly applied to address (i) and (ii), respectively, when the entire players cooperate, referred to as the grand coalition. A recent paper by Misra et al. [3] demonstrates that the Shapley value approach is a promising payoff mechanism to provide right incentives for cooperation in a single-provider peer-assisted service under mild assumptions.

However, in practice, the Internet consists of multiple content providers, even if only giant providers are counted. The focus of our paper is to study the cooperation incentives for multiple providers. In the multi-provider case, the model clearly becomes more complex, thus even classical analysis adopted in the single-provider case becomes much more challenging, and moreover the results and their implications may experience drastic changes. To motivate further, see an example in Fig. 1 with two providers (Google TV and iTunes) and consider two cases of cooperation: (i) separated, where there exists a fixed partition of peers for each provider, and (ii) coalescent, where each peer is possible to assist any provider[1]. In the separated case,
a candidate payoff scheme is based on the Shapley value in each separated coalition. Similarly, in the coalescent case, the Shapley value is also a candidate payoff scheme after the worth function of the grand coalition $N$ (the player set) is defined appropriately. A reasonable definition of the worth function can be the total cost reduction generated by $N$ which is maximized over all combinations of peer partitions to each provider. Then, it is not hard to see that the cost reduction for the coalescent case exceeds that for the separated case, unless the two partitions are equivalent in both cases. This implies that at least one individual in the separated case is underpaid than in the coalescent case under the Shapley-value based payoff mechanism. Thus, providers and users are recommended to form the grand coalition and be paid off based on the Shapley value, i.e., the due desert.

However, it is still questionable whether peers will stay in the grand coalition and thus the consequent Shapley-value based payoff mechanism is desirable in the multi-provider setting. In this paper, we anatomize incentive structures in peer-assisted services with multiple content providers and focus on stability issues from two different angles: stability at equilibrium of Shapley value and convergence to the equilibrium.

Our main contributions are summarized as follows:

1) We first provide a closed-form formula of the Shapley value for a general case of multiple providers and peers. To that end, we define a worth function to be a maximum total cost reduction over all possible peer partitions to each provider. Due to the intractability of analytical computation of the Shapley value, we take a fluid-limit approximation that assumes a large number of peers and re-scales the system with the number of peers. This is a non-trivial generalization of the Shapley value for the single-provider case in [3]. In fact, our formula in Theorem 1 establishes the general Shapley value for distinguished multiple atomic players and infinitesimal players in the context of the Aumann-Shapley (A-S) prices [4] in coalition game theory.

2) We prove that the Shapley value for the multi-provider case is not in the core under mild conditions, e.g., each provider’s cost function is concave. This is in stark contrast to the single-provider case where the concave cost function stabilizes the equilibrium. We also show that, irrespective of stability of the grand coalition, there always exist initial states which are not convergent to the equilibrium. An interesting fact from this part of study is that peers and providers have opposite cooperative preferences, i.e., peers prefer to cooperate with more providers, whereas providers prefer to be separated from other providers.

The insight that our results provide us is the impossibility of cooperation in peer-assisted services with multiple providers. In conjunction with the main contributions mentioned above, we conclude the paper by presenting three examples for non-cooperation among providers: (i) the peers are underpaid than the Shapley payoff, (ii) a provider with more “advantageous” cost function monopolizes all peers, and (iii) Shapley value for each coalition gives rise to an oscillatory behavior of coalition structures. These examples suggest that the system with the separated providers may be unstable as well as unfairness in a peer-assisted service market.
II. Preliminaries

Since this paper deals with multiple content providers and thus a peer can choose any provider to assist, we define a coalition game with a partition (coalition structure), and introduce the payoff mechanisms there.

A. Game with Coalition Structure

A game with coalition structure is a triple \((N,v,P)\) where \(N\) is a player set and \(v:2^N \rightarrow \mathbb{R}\) (\(2^N\) is the set of all subsets of \(N\)) is a worth function, \(v(\emptyset) = 0\). \(v(K)\) is called the worth of a coalition \(K \subseteq N\). \(P\) is called a coalition structure for \((N,v)\); it is a partition of \(N\) where \(P(i) \subseteq P\) denotes the coalition containing player \(i\). The grand coalition is the partition \(P = \{N\}\). For instance, a partition of \(N = \{1, 2, 3, 4, 5\}\) is \(P = \{\{1, 2\}, \{3, 4, 5\}\}\), \(P(4) = \{3, 4, 5\}\), and the grand coalition is \(P = \{\{1, 2, 3, 4, 5\}\}\). \(P(N)\) is the set of all partitions of \(N\). For notational simplicity, a game without coalition structure \((N,v,\{N\})\) is denoted by \((N,v)\). A value of player \(i\) is an operator \(\phi_i(N,v,P)\) that assigns a payoff to player \(i\).

To conduct the equilibrium analysis of coalition games, the notion of core has been extensively used to study the stability of the grand coalition \(P = \{N\}\):

**Definition 1 (Core)** The core is defined as \(\{\phi(N,v) \mid \sum_{i \in N} \phi_i(N,v) = v(N)\) and \(\sum_{i \in K} \phi_i(N,v) \geq v(K), \forall K \subseteq N\}\).

If a payoff vector \(\phi(N,v)\) lies in the core, no player in \(N\) has an incentive to split off to form another coalition \(K\) because the worth of the coalition \(K\), \(v(K)\), is no more than the payoff sum \(\sum_{i \in K} \phi_i(N,v)\). Note that the definition of the core hypothesizes that the grand coalition is already formed ex-ante. We can see the core as an analog of Nash equilibrium from noncooperative games. Precisely speaking, it should be viewed as an analog of strong Nash equilibrium where no arbitrary coalition of players can create worth which is larger than what they receive in the grand coalition. If a payoff vector \(\phi_N(N,v)\) lies in the core, then the grand coalition is stable with respect to any collusion to break the grand coalition.

B. Shapley Value and Aumann-Drèze Value

We provide here the original version \(^5\) of the axiomatic characterization the Shapley value.

**Axiom 1 (Coalition Efficiency, CE)** For all \(i \in N\), \(\sum_{j \in P(i)} \phi_j(N,v,P) = v(P(i))\).

**Axiom 2 (Coalition Restricted Symmetry, CS)** If \(j \in P(i)\) and \(v(K \cup \{i\}) = v(K \cup \{j\})\) for all \(K \subseteq N \setminus \{i, j\}\), then \(\phi_i(N,v,P) = \phi_j(N,v,P)\).

**Axiom 3 (Additivity, ADD)** \(\phi_i(N,v+v',P) = \phi_i(N,v,P) + \phi_i(N,v',P)\) for all coalition functions \(v, v'\) and \(i \in N\).

**Axiom 4 (Null Player, NP)** If \(v(K \cup \{i\}) = v(K)\) for all \(K \subseteq N\), then \(\phi_i(N,v,P) = 0\).

Recall that the basic premise of the Shapley value is that the player set is not partitioned, i.e., \(P = \{N\}\). It is well-known \(^5\), \(^6\) that the Shapley value \(^5\), defined in \(^1\), is uniquely characterized by CE, CS, ADD and NP all for \(P = \{N\}\) as follows:

\[
\phi_i(N,v) = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!(|N| - |S| - 1)!}{|N|!} (v(S \cup \{i\}) - v(S)).
\]

Shapley \(^5\) gives the following interpretation: “(i) Starting with a single member, the coalition adds one player at a time until everybody has been admitted. (ii) The order in which players are to join is determined by chance, with all arrangements equally probable. (iii) Each player, on his admission, demands and is promised the amount which his adherence contributes to the value of the coalition.” The Shapley value quantifies the above that is axiomized (see \(^5\) for the details of the axioms) and has been treated as a worth distribution scheme. The beauty of the Shapley value lies in that the payoff “summarizes” in one number all the possibilities of each player’s contribution in every coalition structure.

Given a coalition structure \(P \neq \{N\}\), one can obtain the Aumann-Drèze value of player \(i\) by taking \(P(i)\), which is the coalition containing player \(i\), to be the player set and by computing the Shapley value of player \(i\) of the reduced game \((P(i), v_{P(i)})\). From this definition, it is easy to see that the A-D value \(\phi\) can be construed as a direct extension of the Shapley value to a game with coalition structure. Formally, it is proven in \(^7\) that the A-D value is uniquely characterized by CE, CS,

\(^2\) A player \(i\) is an element of a coalition \(C = P(i)\), which is in turn an element of a partition \(P\). Lastly, \(P\) is an element of \(P(N)\) while a subset of \(2^N\).
where

\[ \hat{v}(C) = \frac{1}{\phi(C)} \sum_{\mathcal{P} \in \mathcal{P}} v(C \cup \mathcal{P}) - \sum_{\mathcal{P} \subseteq \mathcal{P}} v(C \cup \mathcal{P}) \]

ADD and NP (Axioms [14]). Note that these four axioms uniquely characterize the Shapley value if \( \mathcal{P} = \{ N \} \). Typically in the literature, e.g., [3], [9], the A-D value has been used to analyze the static games where the coalition structure is exogenously given.

Remarkably, it follows from the definition of the A-D value, the payoffs of a coalition \( C \) are affected neither by the player set \( N \) nor other coalitions \( C \in \mathcal{P}, C \neq \mathcal{P}(i) \) as long as the form of the coalition is given. This independence implies the following weaker one:

**Definition 2 (Coalition Independent, CI)** A value is coalition independent if \( \mathcal{P}(i) = \mathcal{P}'(i) \Rightarrow \varphi_1(N, v, \mathcal{P}) = \varphi_1(N, v, \mathcal{P}') \).

Thanks to the independence of the A-D value, in order to decide the payoffs of a game with general coalition structure \( \mathcal{P} \), it suffices to decide the payoffs of players within each coalition, say \( C \in \mathcal{P} \), without considering other coalitions \( C \in \mathcal{P}, C \neq \mathcal{P}(i) \). That is to say, once we decide the payoffs of a coalition \( C \in \mathcal{P} \), the payoffs remains unchanged even though other coalitions, \( C' \in \mathcal{P}, C' \neq C \), vary. It is important to understand that, whereas there are many types of coalition structures \( \mathcal{P} \), there are only two types of coalition \( C \in \mathcal{P} \), (i) a provider with peers or (ii) two or more providers with peers, as depicted in Fig. 1. This also implies that it is sufficient to analyze the two cases in Fig. 1.

**III. Coalition Game in Peer-Assisted Services**

In this section, we first define a coalition game in a peer-assisted service with multiple content providers by classifying the types of coalition structures as separated, where a coalition includes only one provider, and coalescent, where a coalition is allowed to include more than one providers (see Fig. 1). To define the coalition game, we will define a worth function of an arbitrary coalition \( S \subseteq N \) for such two cases. The key message of this section is that the rational behavior of the providers makes the Shapley value approach unworkable because the major premise of the Shapley value, the grand coalition, is not formed in the multi-provider games.

**A. Worth Function in Peer-Assisted Services**

Assume that players \( N \) are divided into two sets, the set of content providers \( Z := \{ p_1, \cdots, p_c \} \), and the set of peers \( H := \{ n_1, \cdots, n_x \} \), i.e., \( N = Z \cup H \). We also assume that the peers are homogeneous, e.g., the same computing powers, disk cache sizes, and upload bandwidths. Later, we discuss that our results can be readily extended to nonhomogeneous peers. The set of peers assisting providers is denoted by \( \bar{H} := \{ n_1, \cdots, n_x, \eta \} \) where \( x = |\bar{H}|/\eta \), i.e., the fraction of assisting peers. We define the worth of a coalition \( S \) to be the amount of cost reduction due to distribution of the contents with the players in \( S \) in both separated and coalescent cases. As discussed in Section II-B to analyze multiple provider cases, it suffices to consider only two cases in Fig. 1. We note that some subsets \( S \subseteq N \) are worthless:

**Definition 3 (Profitable Player Set)** A subset \( S \subseteq N \) is called profitable if \( S \) contains at least one provider and one peer. Adopting this definition, we make a natural assumption that the worth \( \hat{v}(S), S \subseteq N, \) is zero if \( S \) is not profitable.

**Separated case**: Denote by \( \Omega_p^\eta(x) \) the operational cost of a provider \( p \) when the coalition \( S \) consists of \( \eta \) peers and \( x \) fraction of assisting peers. Since the operational cost cannot be negative, we assume \( \Omega_p^\eta(x) > 0 \). Note that from the homogeneity assumption of peers, the cost function depends only on the number of peers and the fraction of assisting peers. Then, we define the worth function \( \hat{v}(S) \) for the coalition \( S \) as:

\[ \hat{v}(S) := \Omega_p^\eta(0) - \Omega_p^\eta(x) \]

where \( \Omega_p^\eta(0) \) corresponds to the cost when there are no assisting peers.

**Coalescent case**: In contrast to the separated case, where a coalition includes a single provider, the worth for the coalescent case is not clear yet, since depending on which peers assist which providers the amount of cost reduction may differ. One of reasonable definitions would be the maximum worth out of all peer partitions, i.e., the worth for the coalescent case is defined by:

\[ v(S) = \max \left\{ \sum_{C \in \mathcal{P}} \hat{v}(C) \mid \mathcal{P} \in \mathcal{P}(S) \text{ such that } |Z \cap C| = 1 \text{ for all } C \in \mathcal{P} \right\}. \]

It should be remarked that the analysis in this paper can be easily extended to other cases than those in Fig. 1 due to CI. The only reason we analyze the two cases is to avoid heavy notations and unnecessary complications.
The definition above implies that we view a coalition containing more than one provider as the most productive coalition whose worth is maximized by choosing the optimal partition $\mathcal{P}^*$ among all possible partitions of $S$. Note that (3) is consistent with the definition (2) for $|Z \cap S| \leq 1$, i.e., $v(S) = \tilde{v}(S)$ for $|Z \cap S| \leq 1$.

Three remarks are in order. First, as opposed to (3) where $\tilde{v}\{p\} = \eta R - \Omega_p^\eta(0)$ ($R$ is the subscription fee paid by any peer), we simply assume that $\tilde{v}\{p\} = 0$. Note that, as discussed in [9, Chapter 2.2.1], it is no loss of generality to assume that, initially, each provider has earned no money. In our context, this means that it does not matter how much fraction of peers is subscribing to each provider because each peer has already paid the subscription fee to providers ex-ante.

Second, it is also important to note that we cannot always assume that $\Omega_p^\eta(x)$ is monotonically decreasing because providers have to pay the electricity expense of the computers and the maintenance cost of the hard disks of assisting peers. For example, a recent study [10] found that Annualized Failure Rate (AFR) of hard disk drives is over 8.6% for three-year old ones. We discuss in Appendix A that, if we consider a more intelligent coalition, the worth function is always non-increasing. However, to simplify the exposition, we assume in this paper that $\Omega_p^\eta(x)$ is non-increasing in $x$ for all $p \in Z$.

Third, the worth function in peer-assisted services can reflect the diversity of peers. It is not difficult to extend our result to the case where peers belong to distinct classes. For example, peers may be distinguished by different upload bandwidths and different hard disk cache sizes. A point at issue for the multiple provider case is whether peers who are not subscribing to the content of a provider may be allowed to assist the provider or not. On the assumption that the content is ciphered and not decipherable by the peers who do not know its password which is given only to the subscribers, providers will allow those peers to assist the content distribution. Otherwise, we can easily reflect this issue by dividing the peers into a number of classes where each class is a set of peers subscribing to a certain content.

### B. Fluid Aumann-Drèze Value for Multiple-Provider Coalitions

So far we have defined the worth of coalitions. Now let us distribute the worth to the players for a given coalition structure $\mathcal{P}$. Recall that the payoffs of players in a coalition are independent from other coalitions by the definition of A-D payoff. Pick a coalition $C$ without loss of generality, and denote the set of providers in $C$ by $\bar{Z} \subset Z$. With slight notational abuse, the set of peers assisting $\bar{Z}$ is denoted by $\bar{H}$. Once we find the A-D payoff for a coalition consisting of arbitrary provider set $\bar{Z} \subset Z$ and assisting peer set $\bar{H} \subset H$, the payoffs for the separated and coalescent cases in Fig. 1 follow from the substitutions $\bar{Z} = Z$ and $\bar{Z} = \{p\}$, respectively. In light of our discussion in Section II-B it is more reasonable to call a payoff mechanism ’A-D payoff’ and ’Shapley payoff’ respectively for the partitioned and non-partitioned games $(N, v, \{\bar{Z} \cup \bar{H}, \cdots\})$ and $(N, v, \{Z \cup H\})$.

**Fluid Limit**: We adopt the limit axioms for large population of users to overcome the computational hardness of the A-D payoffs:

$$
\lim_{\eta \to \infty} \tilde{\Omega}_p^\eta(\cdot) = \tilde{\Omega}_p(\cdot) \quad \text{where} \quad \tilde{\Omega}_p^\eta(\cdot) = \frac{1}{\eta} \tilde{\Omega}_p^\eta(\cdot)
$$

which is the asymptotic operational cost per peer in a very large system. We drop superscript $\eta$ from notations to denote their limits as $\eta \to \infty$. From the assumption $\Omega_p^\eta(x) > 0$, we have $\tilde{\Omega}_p(x) \geq 0$. To avoid trivial cases, we also assume $\tilde{\Omega}_p(x)$ is not constant in the interval $x \in [0, 1]$ for any $p \in Z$. We also introduce the payoff of each provider per user, defined as $\tilde{\varphi}_p^\eta := \frac{1}{\eta} \varphi_p^\eta$.

We now derive the fluid limit equations of the payoffs which can be obtained as $\eta \to \infty$. The proof of the following theorem is given in Appendix III.

**Theorem 1 (A-D Payoff for Multiple Providers)** As $\eta$ tends to infinity, the A-D payoffs of providers and peers under an arbitrary coalition $C = \bar{Z} \cup \bar{H}$ converge to the following equation:

$$
\left\{ \begin{array}{ll}
\tilde{\varphi}_p^\eta(x) = \tilde{\Omega}_p(0) - \sum_{S \subseteq \bar{Z} \setminus \{p\}} \int_0^1 u^{|S|(1-u)^2} \left( M_{\bar{H}}^{\eta}(ux) - M_{\bar{H}}^{\eta}(ux) \right) du, & \text{for } p \in \bar{Z} \\
\tilde{\varphi}_n^\eta(x) = - \sum_{S \subseteq \bar{Z}} \int_0^1 u^{|S|(1-u)^2} \int_0^x \frac{dM_S}{dx}(ux) du, & \text{for } n \in \bar{H}.
\end{array} \right.
$$

Here $M_S^{\eta}(x) := \min \left\{ \sum_{i \in S} \tilde{\Omega}_i(y_i) \mid \sum_{i \in S} y_i \leq x, \ y_i \geq 0 \right\}$ and $M_{\bar{H}}^{\eta}(x) := 0$. Note that $M_{\bar{H}}^{\eta}(x) = \tilde{\Omega}_p(x)$.

On the contrary, the term ‘Shapley payoff’ was used in [3] to refer to the payoff for the game $(N, v, \{\bar{Z} \cup \bar{H}, \cdots\})$ where a proper subset of the peer set assists the content distribution.
Corollary 1 (A-D Payoff for Single Provider) As \( \eta \) tends to infinity, the A-D payoffs of providers and peers who belong to a single-provider coalition, i.e., \( \tilde{Z} = \{ p \} \), converge to the following equation:

\[
\varphi^{(p)}_\pi(x) = \tilde{\Omega}_\pi(0) - \int_0^1 uM^{(p)}_\Omega(ux) du, \\
\varphi^{(p)}_\gamma(x) = -\int_0^1 u \frac{dM^{(p)}_\Omega}{dx}(ux) du, \quad \text{for } n \in \tilde{H}.
\] (6)

Once again, the following corollary for two-provider case is derived from Theorem 1.

Corollary 2 (A-D Payoff for Dual Providers) As \( \eta \) tends to infinity, the A-D payoffs of providers and peers who belong to a dual-provider coalition, i.e., \( \tilde{Z} = \{ p, q \} \), converge to the following equation:

\[
\begin{align*}
\varphi^{(p,q)}_\pi(x) &= \tilde{\Omega}_\pi(0) - \int_0^1 uM^{(p,q)}_\Omega(ux) du - \int_0^1 (1-u)M^{(p)}_\Omega(ux) du + \int_0^1 uM^{(q)}_\Omega(ux) du, \\
&\quad \text{for } n \in \tilde{H}.
\end{align*}
\] (7)

Remark 1 [11] Our result is remarkable in that it establishes the A-D values for distinguished multiple atomic players (the providers) and infinitesimal players (the peers), in the context of the Aumann-Shapley (A-S) prices [4] in coalition game theory.

[11] Lengthy and complicated proof notwithstanding, our formula for the peers can be interpreted easily. Take the second line of (7) as an example. Recall the definition of the Shapley value [11]. The payoff of player \( i \) is the marginal cost reduction \( v(S \cup \{ i \}) - v(S) \) that is averaged over all equally probable arrangements, i.e., the orders of players. It is also implied by [11] that the expectation of the marginal cost is computed under the assumption that the events \(| S | = u \) and \(| S | = u' \) for \( u \neq u' \) are equally probable, i.e., \( P(|S| = u) = P(|S| = u') \). Therefore, in our context of infinite player game in Theorem 1 for every values of \( u \) along the interval \([0, x]\), the subset \( S \subseteq \tilde{Z} \cup \tilde{H} \) contains \( x \) fraction of the peers and the probability that each provider is a member of \( S \) is simply \( u \). Note here that, because the size of peers in \( S, \eta u \), is infinite as \( \eta \to \infty \), the number of providers in \( S \) does not affect the size of \( S \). Therefore, the marginal cost reduction of each peer on the condition that both providers are contained in \( S \) becomes \( u^2 \frac{dM^{(p,q)}_\Omega}{dx}(ux) \). Likewise, the marginal cost reduction of each peer on the condition that only one provider is in the coalition is \( u(1-u) \frac{dM^{(p)}_\Omega}{dx}(ux) \).

C. Stability of the Grand Coalition

The stability guarantee of a payoff vector has been an important topic in coalition game theory. For the single provider case, \(|Z| = 1\), it was shown in [3, Theorem 4.2] that, if the cost function is decreasing and concave, the Shapley incentive structure lies in the core of the game. Plainly speaking, it implies, on the assumption that the grand coalition is formed, there is no provider that both providers are contained in \( S \). As \( \tilde{S} \) contains \( x \) fraction of the peers and infinitesimal players, it is remarkable in that it establishes the A-D values for distinguished multiple atomic players (the providers) and infinitesimal players (the peers). The proof is given in Appendix C.
Theorem 2 (Shapley Payoff for Multiple Providers Not in the Core) If there exists a noncontributing provider, the Shapley payoff for the game \((Z \cup H, v)\) does not lie in the core.

It follows from Lemma 1 that, if all operational cost functions are concave and \(|Z| \geq 2\), the Shapley payoff does not lie in the core, either.

Remark 2 This result appears to be in best agreement with our usual intuition. If there is a provider who does not contribute to the coalition at all in the sense of and is still being paid due to her potential for imaginary contribution assessed by the Shapley formula, which is not actually exploited in the current coalition, other players will agree to expel her to improve their payoffs.

\(\text{Remark 2}^\text{2} \) It is essential to understand that the definition of the core hypothesizes that the grand coalition is already formed \textit{ex-ante}. That is, the notion of the core, which can be viewed as an analog of the strong Nash equilibrium from noncooperative games, assumes that providers and peers are in cooperation (or in the equilibrium). If a payoff vector lies in the core, once they are in cooperation, they will continue to do so. However, whether such a cooperation is ever reached from any initial state or not is not treated by the notion of the core.

The condition \(|Z| \geq 2\) plays an essential role in the theorem. For \(|Z| \geq 2\), the concavity of the cost functions leads to the Shapley value not lying in the core, whereas, for the case \(|Z| = 1\), the concavity of the cost function is proven to make the Shapley incentive structure lie in the core [3, Theorem 4.2].

D. Convergence to the Grand Coalition

The notion of the core lends itself to the stability analysis of the grand coalition on the assumption that the players are already in the equilibrium, \textit{i.e.}, the grand coalition. Theorem 2 raises a point open to further discussion due to the assumption of concave cost function, \textit{e.g.}, for the cost functions that are not concave, it is possible that the Shapley value is in the core. We present that such cases are unlikely to occur by showing that the grand coalition is not a global attractor under some conditions.

To study the convergence of a game with coalition structure to the grand coalition, we define the stability of a game with coalition structure.

Definition 5 (Stable Coalition Structure) We say that a coalition structure \(P'\) blocks \(P\), where \(P', P \in P(N)\), with respect to \(\varphi\) if and only if there exists some \(C \in P'\) such that \(\varphi_i(N, v, \{C, \cdot \cdot \cdot \}) > \varphi_i(N, v, P)\) for all \(i \in C\). In this case, we also say that \(C\) blocks \(P\). If there does not exist any \(P'\) which blocks \(P\), \(P\) is called stable.

It is also important to note that, due to the coalition independence of the A-D value, all stability notions defined by Hart and Kurz coincide with the above simplistic definition.

It is no loss of generality to define the stability without considering the other partitions than \(C\) because the A-D value discussed so far in paper is coalition independent (Definition 2). \textit{i.e.}, the payoff of a player \(i\) does not depend on the forms of other coalitions \(C \neq P(i), C \in P\). The above definition can be intuitively interpreted that, if there exists any subset of players \(C\) who improve their payoffs away from the current coalition structure, they \textit{will} form a new coalition \(C\). In other words, if a coalition structure \(P\) has any blocking coalition \(C\), some rational players will break \(P\) to increase their payoffs.

Unsurprisingly, if a payoff vector lies in the core, the grand coalition is stable in the above sense. This reminds us that the core is about the stability of a particular equilibrium, \textit{i.e.}, the grand coalition. The basic premise here is that players are not clairvoyant, \textit{i.e.}, they are interested only in improving their instant payoffs.

Theorem 3 (A-D Payoff for Multiple Providers Does Not Lead to the Grand Coalition) Suppose \(|Z| \geq 2\) and \(\Omega_p(y)\) is not constant in the interval \(y \in [0, x]\) for any \(p \in Z\) where \(x = |H|/|H|\). The followings hold for all \(p \in Z\) and \(n \in H\).

- The A-D payoff for provider \(p\) in coalition \(\{p\} \cup \bar{H}\) is larger than that in all coalition \(T \cup \bar{H}\) for \(\{p\} \subseteq T \subseteq Z\).
- The A-D payoff of peer \(n\) in coalition \(\{p\} \cup \bar{H}\) is smaller than that in all coalition \(T \cup \bar{H}\) for \(\{p\} \subseteq T \subseteq Z\).

In plain words, a provider, who is in cooperation with a peer set, will receive the highest dividend when she cooperates only with the peers excluding other providers whereas each peer wants to cooperate with as many as possible providers.

Remark 3 It is surprising that, for the multiple provider case, \textit{i.e.}, \(|Z| \geq 2\), each provider benefits from forming the single-provider coalition \textit{whether} the cost function is concave \textit{or not}. There is no \textit{positive} incentives for providers to cooperate
with each other under the implementation of A-D payoffs. On the contrary, a peer always loses by leaving the grand coalition.

R3.2 Why do the multiple providers behave noncooperative?: It is important to observe that this theorem is implied by
\[
\bar{\Omega}_p(0) - M_\Omega(p; x) \geq \bar{\Omega}_p(0) - \left( M_\Omega(T \cup \{p\}) - M_\Omega(T) \right)
\]
which means that the reduction of the operational cost incurred by provider \( p \) to the coalition \( \{p\} \cup \bar{H} \) where \( |\bar{H}|/\eta = x \), or simply the contribution of \( p \) to the coalition, is larger than that to any coalition \( T \cup \bar{H} \) such that \( T \subseteq Z \). Under the implementation of A-D payoff, every provider can contribute more by forming a single-provider coalition, which turns out to be the best strategy for a provider to pursue, hence she is paid more.

R3.3 Recall that Shapley values, \( \tilde{\varphi}^Z_p(1) \) and \( \tilde{\varphi}^Z_n(1) \), are the balanced and comprehensive assessment of players in the game. This theorem states that the grand coalition is not formed so that peers are less paid than their due amounts. Thus the game leads to unfair distribution of the coalition worth.

Upon the condition that each provider begins with a single-provider coalition with a large number of peers, one cannot reach the grand coalition because those single-provider coalitions are already stable in the sense of the stability in Definition 5. That is, the grand coalition is not the global attractor.

IV. A CRITIQUE OF THE A-D PAYOFF FOR SEPARATE PROVIDERS

The discussion so far has centered on the stability of the grand coalition. The result in Theorem 2 suggests that if there is a noncontributing (free-riding) provider, the grand coalition will be broken. The situation is aggravated by Theorem 3 stating that single-provider coalitions will persist if providers are rational. In this section, on the major premise that the providers are separated, we illustrate weak points of the A-D payoff with a few representative examples.

A. Unfairness and Monopoly

Example 1 (Unfairness) Suppose that there are two providers, i.e., \( Z = \{p, q\} \), with \( \bar{\Omega}_p(x) = 2(x - 1)^2/3 + 1/3 \) and \( \bar{\Omega}_q(x) = 1 - x \), both of which are decreasing and convex. When the two providers are separated, from Corollary 1 we have \( \tilde{\varphi}^Z_p(x) = -2x^2/9 + 2x/3, \tilde{\varphi}^Z_q(x) = x/2, \tilde{\varphi}^Z_n(x) = -4x/9 + 2/3 \) and \( \tilde{\varphi}^Z_{p,n}(x) = 1/2 \). Shapley values of providers and peers can be obtained from Corollary 2. All values are shown in Fig. 2 as functions of \( x \). In line with Theorem 2 providers are paid more than their Shapley values, whereas peers are paid less than theirs. As discussed in Section I and Remark 3 the Shapley values can be interpreted as the balanced expectations of the marginal contribution of each player to the whole peer-assisted system. On the contrary, the definition of A-D payoff does not take into account the Shapley value of players. Therefore, we can see that the A-D payoff is unfair in this sense.

Coming back to Fig. 2 we can see that each peer \( n \) will be paid 2/3 (\( \tilde{\varphi}^Z_{p,n}(0) \)) when he is contained by the coalition \( \{p, n\} \) and the payoff decreases with the number of peers in this coalition. On the other hand, provider \( p \) wants to be assisted by as many peers as possible because \( \tilde{\varphi}^Z_{p,n}(x) \) is increasing in \( x \). If it is possible for \( n \) to prevent other peers from joining the
coalition, he can get 2/3. However, it is more likely that no peer can kick out other peers. Thus, \( p \) will be assisted by \( x = 3/8 \) fraction of peers, which is the unique solution of \( \tilde{\varphi}_n^{[p]}(x) = \tilde{\varphi}_n^{[q]}(x) \) while \( q \) assisted by \( 1 - x = 5/8 \) fraction of peers.

**Example 2 (Monopoly)** Consider a two-provider system \( Z = \{p, q\} \) with \( \tilde{\Omega}_p(x) = 1 - x^{3/2} \) and \( \tilde{\Omega}_q(x) = 1 - 2x/3 \), both of which are decreasing and concave. Similar to Example 1, we can obtain \( \tilde{\varphi}_n^{[p]}(x) = 2x^{3/2}/5, \tilde{\varphi}_n^{[q]}(x) = x/3, \) \( \tilde{\varphi}_n^{[pq]}(x) = 3x^{1/2}/5 \) and \( \tilde{\varphi}_n^{[q]}(x) = 1/3 \). All values including the Shapley values are shown in Fig. 3. Not to mention unfairness in line with Theorem 5, provider \( p \) monopolizes the whole peer-assisted services. No provider has an incentive to cooperate with other provider and each peer has to choose between the two providers. It can be seen that all peer will assist the content distribution of \( p \) because \( \tilde{\varphi}_n^{[p]}(x) > \tilde{\varphi}_n^{[q]}(x) \) for \( x > 25/81 \). Appealing to Definition 3 if the providers are initially separated, the coalition structure will converge to the service monopoly by \( p \). In line with Lemma 1 and Theorem 2 even if the grand coalition is supposed to be the initial condition, it is not stable in the sense of the core. The noncontributing provider (Definition 4) in this example is \( q \).

**B. Instability of A-D Payoff Mechanism**

The last example illustrates the A-D payoff can even induce an analog of the limit cycle in nonlinear control theory. The oscillatory behavior of the A-D payoff was studied only recently by Tutic [13] who showed that, in any game with less than four players, \( |N| = |Z \cup H| \leq 3 \), there exists a stable coalition structure with respect to the A-D value. This implies that we need at least four players so that such oscillatory phenomena occur. However, since we have additional constraints on the worth function from Definition 3 (profitable player set), our games are slightly different from those in [13].

**Example 3 (Oscillation)** Consider a game with two providers and two peers where \( N = \{p_1, p_2, n_1, n_2\} \). If \( \{n_1\}, \{n_2\} \) and \( \{n_1, n_2\} \) assist the content distribution of \( p_1 \), the reduction of the distribution cost is respectively 10$, 9$ and 11$ per month. However, the hard disk maintenance cost incurred from a peer is 5$. In the meantime, if \( \{n_1\}, \{n_2\} \) and \( \{n_1, n_2\} \) assist the content distribution of \( p_2 \), the reduction of the distribution cost is respectively 6$, 3$ and 13$ per month. In this case, the hard disk maintenance cost incurred from a peer is supposed to be 2$ due to smaller contents of \( p_2 \) as opposed to those of \( p_1 \). We can compute the net cost reduction for all possible coalitions. For example, if \( n_1 \) and \( n_2 \) help \( p_1 \), the coalition worth becomes \( v(\{n_1, n_2, p_1\}) = 11 - 5\cdot5 - 5\cdot5 = 1\).

Using the same expression 3 as in Section 11 it is easy to see that the coalition worths for coalescent provider cases are \( v(\{p_1, p_2, n_1\}) = 5, v(\{p_1, p_2, n_2\}) = 4 \) and \( v(\{p_1, p_2, n_1, n_2\}) = 9 \). Since it is very tedious to compute the A-D payoffs for all coalition structures and to determine their stability, we refer to Appendix E for a detailed analysis. For notational simplicity, we adopt a simplified expression for coalitional structure \( P \): A coalition \( \{a, b, c\} \in P \) is denoted by \( abc \) and each singleton set \( \{i\} \) is denoted by \( i \). We first observe that the Shapley payoff of this example does not lie in the core. Suppose that peers continue to form a new coalition \( C' \) when they can improve away from the current coalition \( C \). That is, if there is a blocking coalition \( C' \) (Definition 5), they will betray \( C \). It is easy to see from Appendix E that almost all coalition structures are transient (or have a blocking coalition) and only four coalition structures are recurrent (or do not have any blocking coalition).
in the sense that they have finite hitting times with probability one.

As time tends to infinity, the A-D payoff exhibits an oscillation of the partition \( P \) consisting of the four recurrent coalition structures as shown in Fig. 4. We can have a better understanding of this game from this figure. For instance, let us begin with the partition \( \{p_1, p_2n_1n_2\} \). Player \( p_1 \) could have achieved the maximum payoff if it had formed a coalition only with \( n_1 \). However, player \( n_1 \) will remain in the current coalition because he does not improve away from the current coalition. Player \( p_1 \) makes a cat’s paw of \( n_2 \) to break the coalition \( p_2n_1n_2 \) so that he can form coalition \( p_1n_2 \). As soon as the coalition \( p_2n_1n_2 \) is broken, \( p_1 \) betrays \( n_2 \) to increase his payoff by colluding with \( n_1 \). As of now, from the state-of-the-art in the literature on this behavior, it is not yet clear how this behavior will be developed in large-scale systems.

V. CONCLUDING REMARKS

A quote from an interview of BBC iPlayer with CNET UK [14]: “Some people didn’t like their upload bandwidth being used. It was clearly a concern for us, and we want to make sure that everyone is happy, unequivocally, using iPlayer.”

In this paper, we have studied whether the Shapley incentive structure in peer-assisted services would be in conflict with the pursuit of profits by rational content providers and peers. A lesson from our analysis is summarized as: Even though it is righteous to pay peers more because they become relatively more useful as the number of peer-assisted services increases, the content providers will not admit that peers should receive their due deserts. The providers tend to persist in single-provider coalitions. In the sense of the classical stability notion, called ‘core’, the cooperation would have been broken even if we had begun with the grand coalition as the initial condition. Lastly, we have illustrated yet another problems when we use the Shapley-like incentive for the exclusive single-provider coalitions. These results suggest that the profit-sharing system, Shapley value, and hence its fairness axioms, are not compatible with the selfishness of the content providers. We believe that a realistic incentive structure in peer-assisted services should reflect a trade-off between fairness and competition among individuals.

REFERENCES


*Note that, as discussed in [12], one can get two types of resulting coalition structures when a player departs from a coalition. For example, if player \( n_1 \) departs from her coalition in \( \{p_1n_1n_2, p_2\} \) to form a coalition with \( p_2 \), we may get either \( \{p_1, n_2, p_2n_1\} \) or \( \{p_2n_1, p_2n_2\} \). In this paper, we assume only the latter case to simplify the exposition.
A. An Intelligent Coalition for Superadditive Worth Functions

To minimize the operational cost, it is also reasonable to assume that, if the assistance from some peers increases the operational cost, they should not be allowed to assist the content distribution. In light of this, the worth of $S$ can be defined as a maximum of $\hat{v}(S)$:

$$v(S) := \Omega_p^n(0) - \min_{0 \leq y \leq x} \Omega_p^n(y). \quad (9)$$

There is a subtle difference between (2) and (9). Informally speaking, not all players within a coalition will remain passively when some outside peers are to decrease the worth of the coalition. Therefore, in games with worth function (2), peers who can decrease the worth are not even eligible for membership of $S$. Note that (2) may not satisfy the following property unless $\Omega_p^n(x)$ is monotonically decreasing:

**Definition 6 (Superadditivity)** A worth function is superadditive if $(S, T \subseteq N$ and $S \cap T = \emptyset) \Rightarrow v(S \cup T) \geq v(S) + v(T)$. It can be seen that (2) is a non-superadditive function, to which most coalition game theory is not applicable. Very often, the cores of such games are empty (See Definition 1).

For example, for cost function $\Omega_p^n(x) = (x - 1)^2 + 0.5x$ which increases when $x \geq 0.75$, one can show that the core of game $(S, \hat{v})$ where $S \cap \tilde{Z} = \{p\}$ and $|S \cap \tilde{H}|/\eta = x$ is empty by using the Bondareva-Shapley Theorem [9, Theorem 3.1.4]. That is, for any payoff mechanism $\varphi_i(S, \hat{v})$ to distribute the worth $\hat{v}(S)$, there exists a coalition $K \subseteq S$ such that $\varphi_K(S, \hat{v}) < \hat{v}(K)$.

In the meantime, it is implied by (9) that $x - y$ fraction of peers, where $y$ is the solution of the min operation in (2), are not assisting the content distribution, hence the provider does not need to pay the expense incurred by those peers. That is, the members of each coalition is more intelligent than those with (3) so that the worth function is non-increasing in $x$.

B. Proof of Theorem 1

We use notation $\hat{\varphi}^Z(x)$ to denote $\hat{\varphi}(\tilde{Z} \cup \tilde{H}, v)$. We use the mathematical induction to prove this theorem. The equation (5) holds for $|\tilde{Z}| = 0$ and $\tilde{Z} = \emptyset$ (empty set) because we have from (5) that there is no provider to pay and $\hat{\varphi}^n_p(x) = 0$ for $n \in \tilde{H}$.

Now we assume that (5) holds for all $\Xi \subseteq \tilde{Z}$ such that $|\Xi| \leq \xi$ where $\xi \geq 0$. To prove Theorem 1 it suffices to show that (5) also holds for all $\Xi' \subseteq \tilde{Z}$ such that $|\Xi'| = \xi + 1$. To this end, we first apply Axiom CE. As $\eta$ tends to infinity while $x$ remains unchanged, for $p \in \Xi'$ and $n \in \tilde{H}$, Axiom CE for the partition $\{\Xi' \cup \tilde{H}\}$ can be rewritten as follows:

$$\sum_{p \in \Xi'} \hat{\varphi}_p^n(x) + x \hat{\varphi}_n^\Xi(x) = \sum_{p \in \Xi'} \hat{\Omega}_p^n(0) - M_1^\Xi(x) \quad (10)$$

which is the the normalized (which we did in (4)) total coalition worth created by the coalition $\Xi' \cup \tilde{H}$. Another axiom we apply is Axiom FAIR (fairness) which was used by Myerson [15] to characterize the Shapley value. It follows from FAIR that

$$\hat{\varphi}_n^\Xi(x) - \hat{\varphi}_n^\Xi \setminus \{p\}(x) = \frac{d}{dx} \hat{\varphi}_p^n(x), \quad \text{for all } p \in \Xi'. \quad (11)$$

Summing up these equation for $p \in \Xi'$ and dividing the sum by $|\Xi'| = \xi + 1$, we obtain

$$\hat{\varphi}_n^\Xi(x) = \frac{1}{\xi + 1} \sum_{p \in \Xi'} \left( \hat{\varphi}_n^\Xi \setminus \{p\}(x) + \frac{d}{dx} \hat{\varphi}_p^n(x) \right) = \frac{1}{\xi + 1} \sum_{p \in \Xi'} \hat{\varphi}_n^\Xi \setminus \{p\}(x) + \frac{1}{\xi + 1} \sum_{p \in \Xi'} \hat{\varphi}_p^n(x) \quad (12)$$

by plugging which into (10), we obtain

$$(\xi + 1) \sum_{p \in \Xi'} \hat{\varphi}_p^n(x) + x \frac{d}{dx} \sum_{p \in \Xi'} \hat{\varphi}_p^n(x) = (\xi + 1) \sum_{p \in \Xi'} \hat{\Omega}_p^n(0) - (\xi + 1) M_1^\Xi(x) - x \sum_{p \in \Xi'} \hat{\varphi}_n^\Xi \setminus \{p\}(x). \quad (13)$$

Since we know the form of $\hat{\varphi}_n^\Xi \setminus \{p\}(x)$ for all $p \in \Xi'$ from the assumption (\because |\Xi' \setminus \{p\}| = \xi), (13) is an ordinary differential
equation of unknown function $\sum_{p \in \Xi'} \tilde{\varphi}'_p(x)$. Denote the RHS of (13) by $G(x)$. Appealing to [3, Lemma 3], we get

$$\sum_{p \in \Xi'} \tilde{\varphi}'_p(x) = \int_0^1 u^\xi G(u|x)du = \sum_{p \in \Xi'} \tilde{\Omega}_p(0) - \int_0^1 u^\xi (\xi + 1) M^S_{\Omega}(ux)du - \int_0^1 u^\xi+1 x \sum_{p \in \Xi'} \tilde{\varphi}'_{n\setminus\{p\}}(ux)du,$$

$$\frac{d}{dx} \sum_{p \in \Xi'} \tilde{\varphi}'_p(x) = -(\xi + 1) \int_0^1 u^\xi+1 M^S_{\Omega}(ux)du - \int_0^1 u^\xi+1 \sum_{p \in \Xi'} \tilde{\varphi}'_{n\setminus\{p\}}(ux)du - \int_0^1 u^\xi+2 x \sum_{p \in \Xi'} \frac{d \tilde{\varphi}'_{n\setminus\{p\}}}{dx}(ux)du \quad (14)$$

where the last expression follows by integrating the last term of (14) by parts. From (12) and (15), \( \tilde{\varphi}'_{n\setminus\{p\}}(x) \) is rearranged as

$$\tilde{\varphi}'_{n\setminus\{p\}}(x) = - \int_0^1 u^\xi+1 \frac{d M^S_{\Omega}}{dx}(ux)du + \int_0^1 u^\xi+1 \sum_{p \in \Xi'} \tilde{\varphi}'_{n\setminus\{p\}}(ux)du. \quad (16)$$

From the assumption, \( \tilde{\varphi}'_{n\setminus\{p\}}(x) \) is given by (5) for \( \tilde{Z} = \Xi' \setminus \{p\} \), which is plugged into the last term of (16) to yield

$$\int_0^1 u^\xi+1 \sum_{p \in \Xi'} \tilde{\varphi}'_{n\setminus\{p\}}(ux)du = - \sum_{p \in \Xi'} \sum_{S \subseteq \Xi \setminus \{p\}} \int_0^1 \int_0^1 (u - ut)^{\xi - |S|} f(ux)udtdu.$$ \quad (17)

In the meantime, we need the following identity to reduce the double integral of (17):

$$\int_0^1 \int_0^1 (u - ut)^{\xi - |S|} f(ux)udtdu = \int_0^1 \int_0^1 \tau^{|S|} f(ux)udtdu = \int_0^1 \int_0^1 \tau f(ux)udtdu.$$

where we used the change of variable \( ut = \tau \) and changed the order of the double integration with respect to \( u \) and \( \tau \). Plugging (18) into (17) yields

$$\int_0^1 u^\xi+1 \sum_{p \in \Xi'} \tilde{\varphi}'_{n\setminus\{p\}}(ux)du = - \sum_{p \in \Xi'} \sum_{S \subseteq \Xi \setminus \{p\}} \frac{1}{\xi + 1 - |S|} \int_0^1 u^{|S|}(1 - u)^{\xi + 1 - |S|} \frac{d M^S_{\Omega}}{dx}(ux)du$$

$$= - \sum_{S \subseteq \Xi \setminus \{p\}} \int_0^1 u^{|S|}(1 - u)^{\xi + 1 - |S|} \frac{d M^S_{\Omega}}{dx}(ux)du. \quad (19)$$

where the last equality holds because

$$\sum_{p \in \Xi'} \sum_{S \subseteq \Xi \setminus \{p\}} f(S) = \sum_{S \subseteq \Xi \setminus \{p\}} f(S) = (\xi + 1) \sum_{|S| = 1} \left( \frac{\xi}{|S|} \right) = \xi + 1.$$ 

Plugging (19) into (16) establishes the following desired result:

$$\tilde{\varphi}'_n(x) = - \sum_{S \subseteq \Xi} \int_0^1 u^{|S|}(1 - u)^{\xi + 1 - |S|} \frac{d M^S_{\Omega}}{dx}(ux)du \quad (20)$$

from which follows

$$\tilde{\varphi}'_n(x) - \tilde{\varphi}'_{n\setminus\{p\}}(x) = - \sum_{S \subseteq \Xi} \int_0^1 u^{|S|}(1 - u)^{\xi + 1 - |S|} \frac{d M^S_{\Omega}}{dx}(ux)du + \sum_{S \subseteq \Xi \setminus \{p\}} \int_0^1 u^{|S|}(1 - u)^{\xi - |S|} \frac{d M^S_{\Omega}}{dx}(ux)du.$$

Since the first term of the RHS can be decomposed into the following:

$$- \sum_{S \subseteq \Xi \setminus \{p\}} \int_0^1 u^{|S|+1}(1 - u)^{\xi + 1 - (|S|+1)} \frac{d M^S_{\Omega}(p)}{dx}(ux)du - \sum_{S \subseteq \Xi \setminus \{p\}} \int_0^1 u^{|S|}(1 - u)^{\xi + 1 - |S|} \frac{d M^S_{\Omega}}{dx}(ux)du,$$
we can obtain
\[ \phi_n(x) - \phi_n'(p) = - \sum_{S \subseteq \Xi \setminus \{p\}} \int_0^1 u^{[S]}(1 - u)^{[S]} \left( dM_{\Omega}^{S \cup \{p\}}(ux) - dM_{\Omega}^S(ux) \right) \, du. \]  

(21)

Integrating (11) from 0 to \( x \) with respect to \( x \) and from (21), we get
\[ \phi_p(x) = - \sum_{S \subseteq \Xi \setminus \{p\}} \int_0^1 u^{[S]}(1 - u)^{[S]} \left( M_{\Omega}^{S \cup \{p\}}(ux) - M_{\Omega}^S(ux) \right) \, du \]
\[ = \bar{\Omega}_p(0) - \sum_{i \in Z \setminus \{p\}} \int_0^1 u^{i} \left( 1 - u \right)^i \left( M_{\Omega}^{S \cup \{p\}}(ux) - M_{\Omega}^S(ux) \right) \, du, \]

which finally establishes that (5) also holds for all \( \Xi' \subseteq \Xi \) such that \( |\Xi'| = \xi + 1 \), hence completing the proof.

C. Proof of Theorem 6

To prove the theorem, we need to show that the condition for the core in Definition 4 is violated, implying that it suffices to show the following:
\[ \phi_p(Z(1)) > \sum_{i \in Z} \bar{\Omega}_i(0) - M_{\Omega}^Z(1) - \left( \sum_{i \in Z \setminus \{p\}} \bar{\Omega}_i(0) - M_{\Omega}^Z(1) \right), \]  
\[ = \bar{\Omega}_p(0) - \left( M_{\Omega}^Z(1) - M_{\Omega}^Z(1) \right). \]  

(22)

This means that the payoff of \( p \in Z \) is greater than the marginal increase of the limit worth, i.e.,
\[ \lim_{\eta \to \infty} \frac{1}{\eta} v(Z \cup H) - \lim_{\eta \to \infty} \frac{1}{\eta} v((Z \setminus \{p\}) \cup H). \]

Subtracting the RHS of (22) from the LHS of (22) and using the expression of \( \phi_p(Z)(1) \) in (5), we have
\[ M_{\Omega}^Z(1) - M_{\Omega}^Z(1) - \sum_{S \subseteq Z \setminus \{p\}} \int_0^1 u^{[S]}(1 - u)^{[S]} \left( M_{\Omega}^{S \cup \{p\}}(u) - M_{\Omega}^S(u) \right) \, du. \]  

(23)

From the assumption, there exists a noncontributing provider which we denote by \( p \). From Definition 4, we have \( M_{\Omega}^Z(1) - M_{\Omega}^Z(1) = \bar{\Omega}_p(0) \). Plugging this into (23) yields:
\[ \bar{\Omega}_p(0) - \sum_{S \subseteq Z \setminus \{p\}} \int_0^1 u^{[S]}(1 - u)^{[S]} \left( M_{\Omega}^{S \cup \{p\}}(u) - M_{\Omega}^S(u) \right) \, du. \]  

(24)

To show that (24) is strictly positive, we rewrite its integrand as follows:
\[ M_{\Omega}^{S \cup \{p\}}(y) - M_{\Omega}^S(y) = \min \left\{ \sum_{i \in S \cup \{p\}} \bar{\Omega}_i(y_i) \mid \sum_{i \in S \cup \{p\}} y_i \leq y, y_i \geq 0 \right\} - \min \left\{ \sum_{i \in S} \bar{\Omega}_i(y_i) \mid \sum_{i \in S} y_i \leq y, y_i \geq 0 \right\} \]

where the first term in the RHS can be rearranged as
\[ \min \left\{ \sum_{i \in S \cup \{p\}} \bar{\Omega}_i(y_i) \mid \sum_{i \in S \cup \{p\}} y_i \leq y, y_i \leq 0 \right\} \leq \bar{\Omega}_p(0) + \min \left\{ \sum_{i \in S} \bar{\Omega}_i(y_i) \mid \sum_{i \in S} y_i \leq y, y_i \geq 0 \right\} \]

where the inequality holds from that \( \bar{\Omega}_i(y), i \in Z \), are non-increasing. It can be easily seen that the inequality holds by considering two cases \( y_p = 0 \) and \( y_p > 0 \). The inequality becomes strict when \( S = \emptyset \) over some interval in \([0, x]\) whose length is positive due to the assumption that \( \bar{\Omega}_p(y) \) is not constant in the interval \( y \in [0, x] \) and non-increasing. From this, we can see that (24) is greater than
\[ \bar{\Omega}_p(0) - \sum_{S \subseteq Z \setminus \{p\}} \int_0^1 u^{[S]}(1 - u)^{[S]} \bar{\Omega}_p(0) \, du = 0 \]
which establishes (22), hence completing the proof.

D. Proof of Theorem 3

To prove Theorem 3 it suffices to show that the following holds for \( \{p\} \subseteq T \) such that \( T \subseteq Z \):

\[
\varphi_p(x) - \varphi_p^T(x) = \sum_{S \subseteq T \setminus \{p\}} \int_0^1 u^{|S|}(1 - u)^{|T| - |S|} \left( M_{\Omega}^{S \cup \{p\}}(ux) - M_{\Omega}^{S}(ux) \right) du - \int_0^1 M_{\Omega}^{\{p\}}(ux) du > 0
\]

which implies that the payoff of \( p \) when it is the only provider of the coalition is larger than that with other providers \( T \setminus \{p\} \).

To this end, we first observe that, for \( y \)

\[
M_{\Omega}^{S \cup \{p\}}(y) - M_{\Omega}^{S}(y) = \min \left\{ \sum_{i \in S \cup \{p\}} \bar{\Omega}_i(y_i) \mid \sum_{i \in S \cup \{p\}} y_i \leq y, y_i \geq 0 \right\} - \min \left\{ \sum_{i \in S} \bar{\Omega}_i(y_i) \mid \sum_{i \in S} y_i \leq y, y_i \geq 0 \right\}
\]

Here the first term in the RHS can be rearranged as

\[
\min \left\{ \sum_{i \in S \cup \{p\}} \bar{\Omega}_i(y_i) \mid \sum_{i \in S \cup \{p\}} y_i \leq y, y_i \geq 0 \right\} \geq M_{\Omega}^{\{p\}}(y) + \min \left\{ \sum_{i \in S} \bar{\Omega}_i(y_i) \mid \sum_{i \in S} y_i \leq y, y_i \geq 0 \right\}
\]

where the inequality holds from that \( M_{\Omega}^{\{i\}}(y), i \in T \), are non-increasing. It can be easily seen that the inequality holds by considering two cases \( y_p = 0 \) and \( y_p > 0 \). The inequality becomes strict when \( S = \emptyset \) over some interval in \([0, x]\) whose length is positive due to the assumption that \( \Omega_p(y) \) is not constant in the interval \( y \in [0, x] \) and non-increasing. From this inequality, we have \( M_{\Omega}^{S \cup \{p\}}(y) - M_{\Omega}^{S}(y) \geq M_{\Omega}^{\{p\}}(y) \) and the inequality is strict over some interval of positive length. Plugging this relation into (25) yields

\[
\varphi_p(x) - \varphi_p^T(x) = \left( \sum_{S \subseteq T \setminus \{p\}} \int_0^1 u^{|S|}(1 - u)^{|T| - |S|} M_{\Omega}^{\{p\}}(ux) du \right) - \int_0^1 M_{\Omega}^{\{p\}}(ux) du > 0.
\]

Also, we can see the following from (10):

\[
\lim_{\eta \to \infty} v(\{p\} \cup \bar{H})/\eta = \bar{\Omega}_p(0) - M_{\Omega}^{\{p\}}(x) \leq \sum_{i \in T} \bar{\Omega}_i(0) - M_{\Omega}^{\{p\}}(x) = \lim_{\eta \to \infty} v(T \cup \bar{H})/\eta
\]

which, when combined with \( \varphi_p(x) > \varphi_p^T(x) \), implies the second part of the theorem.

E. Computation of the A-D Payoff in Example 3

In this section, we compute the A-D payoff of the Example 3. From the description of the cost reduction and the hard disk maintenance cost incurred from peers in Example 3 we can compute the coalition worth function as follows:

\[
\hat{v}(S) = \begin{cases} 
0, & \text{if } S \text{ is not profitable,} \\
5, & \text{if } S = \{p_1, n_1\}, \\
4, & \text{if } S = \{p_1, n_2\}, \\
1, & \text{if } S = \{p_1, n_1, n_2\}, \\
4, & \text{if } S = \{p_2, n_1\}, \\
1, & \text{if } S = \{p_2, n_2\}, \\
9, & \text{if } S = \{p_2, n_1, n_2\}.
\end{cases}
\]

For notational simplicity, we adopt a simpler expression for coalitional structure \( \mathcal{P} \): A coalition \( \{a, b, c\} \in \mathcal{P} \) is denoted by \( abc \) and each singleton set \( \{i\} \) is denoted by \( i \). Applying these two conventions, we can simplify \( \{\{p_1, n_2\}, \{p_2\}, \{n_1\}\} \) into \( \{p_1n_2, p_2, n_1\} \). Table I contains A-D payoffs and blocking coalitions \( C \subseteq N \) for any coalition structure.
### TABLE I

**Example 3: An Oscillatory Aumann-Drèze Payoff**

<table>
<thead>
<tr>
<th></th>
<th>{p_1, p_2, n_1, n_2}</th>
<th>{p_1, n_1, n_2}</th>
<th>{p_1, p_2, n_1}</th>
<th>{p_1, p_2, n_2}</th>
<th>{p_1, n_2, n_2}</th>
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<tr>
<td>(\varphi_{p_1})</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5/2 = 2.5</td>
</tr>
<tr>
<td>(\varphi_{p_2})</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>1/2 = 0.5</td>
</tr>
<tr>
<td>(\varphi_{n_1})</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5/2 = 2.5</td>
</tr>
<tr>
<td>(\varphi_{n_2})</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1/2 = 0.5</td>
</tr>
<tr>
<td>(C)</td>
<td>{p_1, n_1, n_2}, {p_1, p_2, n_1}, {p_2, n_2}, {p_1, p_2, n_2}, {p_1, n_2, n_2}, {p_2, n_1, n_2}</td>
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<td>{p_1, n_2, n_2}</td>
</tr>
<tr>
<td><strong>recurrent</strong></td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>O</td>
</tr>
<tr>
<td>(\varphi_{p_1})</td>
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<td>7/6 = 1.17</td>
<td>0</td>
<td>5/3 = 1.67</td>
<td>2</td>
</tr>
<tr>
<td>(\varphi_{p_2})</td>
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<td>2/3 = 0.67</td>
<td>23/6 = 3.83</td>
<td>1/6 = 0.17</td>
<td>0</td>
</tr>
<tr>
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<td>19/6 = 3.17</td>
<td>10/3 = 3.33</td>
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<td>0</td>
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<tr>
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<td>0</td>
<td>11/6 = 1.83</td>
<td>13/6 = 2.17</td>
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<tr>
<td>(C)</td>
<td>{p_1, n_2}</td>
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<td>{p_1, n_1, p_2}</td>
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<tr>
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<td>X</td>
<td>O</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>(\varphi_{p_1})</td>
<td>11/6 = 1.83</td>
<td>5/2 = 2.5</td>
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<td>2</td>
<td>0</td>
</tr>
<tr>
<td>(\varphi_{p_2})</td>
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<td>1/2 = 0.5</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>(\varphi_{n_1})</td>
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<td>5/2 = 2.5</td>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>(\varphi_{n_2})</td>
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<td>1/2 = 0.5</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>(C)</td>
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<td>{p_1, n_1, p_2, n_1}, p_2 n_1, p_2 n_2</td>
<td>{p_1, n_1, p_2, n_1}, p_2 n_1, p_2 n_2</td>
<td>{p_1, n_1, p_2, n_1}, p_2 n_1, p_2 n_2</td>
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</tr>
<tr>
<td><strong>recurrent</strong></td>
<td>X</td>
<td>O</td>
<td>X</td>
<td>O</td>
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